Chapter 10 - Day 5

If the velocity $v(t)$ of an object at time t is always positive, then the area under the graph of $v(t)$ and above the t-axis represents the total distance traveled by the object from $t=a$ to $t=b$.

Ex: A train travels along a track and has velocity $v(t) = 76t$ for the first half hour of travel. Its velocity is constant and $v(t) = 76/2$ after the first half hour. Time t is in hours.

- How far does the train travel in the 2nd hour of travel?

$$\int_1^2 v(t)\,dt = \int_1^2 38\,dt$$

$$= (38t)\Big|_1^2$$

$$= 76 - 38 = \boxed{38 \text{ miles}}$$

• How far does the train travel in the 1st hour?

$$\int_0^1 v(t)\,dt = \int_0^{1/2} 76t\,dt + \int_{1/2}^1 38\,dt$$

$$= \left(38t^2\right)\Big|_0^{1/2} + \left(38t\right)\Big|_{1/2}^1$$

$$= 38\left(\tfrac{1}{2}\right)^2 - 38(0)^2 + 38(1) - 38\left(\tfrac{1}{2}\right)$$

$$= 9.5 - 0 + 38 - 19$$

$$= \boxed{28.5 \text{ miles}}$$

<u>Ex</u>: A rock is dropped from a height of 21 ft. Its velocity at time t after its dropped is $v(t) = -32t$ where t is in seconds. How far is the rock from the ground 1 second after its dropped?

let $h(t)$ be the height of the rock.

then $h(0) = 21$ and $h'(t) = v(t) = -32t$

So $h(t) = \int v(t)\,dt = \int -32t\,dt$

$$= -16t^2 + c$$

Since $h(0) = 21$ then $h(0) = -16 \cdot 0^2 + C = 21$

$$\Rightarrow C = 21$$

thus $h(t) = -16t^2 + 21$

lastly, after 1 second, the height is

$$h(1) = -16 \cdot 1^2 + 21 = \boxed{5 \text{ feet}}$$

Ex: An object thrown from a cliff with an initial speed of 5 ft/sec and its speed after t seconds is $s(t) = 10t + 5$. If the object lands after 7 seconds, how high is the cliff?

$$\text{distance traveled} = \int_0^7 (10t+5)\,dt$$

$$= \left(5t^2 + 5t\right)\Big|_0^7$$

$$= 5(7^2) + 5(7) - \left[5(0)^2 + 5(0)\right]$$

$$= \boxed{280 \text{ ft}}$$

Think about when we average a list of numbers....

$$\frac{y_1 + y_2 + y_3 + \ldots + y_n}{n}$$

But what if we are trying to average infinitely many values?

More generally, we can average a function!

The <u>average of a function</u> f on $[a,b]$ is

$$\frac{\int_a^b f(x)\,dx}{b-a}$$

Ex: What is the average of $f(x) = x^2$ over the interval $[0,6]$?

$$\text{avg} = \frac{\int_0^6 x^2\,dx}{6-0} = \frac{\left(\frac{1}{3}x^3\right)\Big|_0^6}{6}$$

$$= \frac{\frac{1}{3}(6)^3 - \frac{1}{3}(0)^3}{6} = \boxed{12}$$